# Consumption-based $CO_2$ Emissions and Carbon Leakage: Results from the Global Resource Accounting Model GRAM

*Version from April 30, 2010 submitted to the*
***International Conference on Economic Modeling (Ecomod 2010)***

Authors: Martin Bruckner (SERI), Stefan Giljum (SERI), Christian Lutz (GWS) and Kirsten S. Wiebe (GWS)

Contact details: GWSmbH, Heinrichstr. 30, 49080 Osnabrück, Germany
Phone +49 541 40933 230, Email wiebe @ gws-os.com

Abstract: Anthropogenic greenhouse gas emissions are a major cause of climate change. Policies in the OECD countries aimed at the reduction of nationally produced $CO_2$ emissions may result in the relocation of emission intensive industries into countries with less stringent policies. This relocation often not only leads to a simple relocation of emissions, but to an absolute increase of emissions due to less advanced technology in the countries of relocation, thus not contributing to climate change mitigation at all. Additionally, newly emerging economies without harsh climate change policies such as China and India produce more goods for domestic consumption as well as for exports than ever before. The OECD countries quintupled their import volume from these countries between 1995 and 2005. This process of relocating industrial plants as well as increasing imports from countries without stringent climate change policies is called carbon leakage. Using the Global Resource Accounting Model (GRAM), we are able to show the extent of carbon leakage between OECD countries and the newly emerging economies Argentina, Brazil, China (including Hong Kong and Taiwan), India, South Africa and Russia (BRICSA) for all years between 1995 and 2005. These findings become particularly relevant, as the externalisation of environmental burden through international trade might be an effective strategy for industrialised countries to maintain high environmental quality within their own borders, while externalising the negative environmental consequences of their consumption processes to other parts of the world.

JEL classification: C67, F18, Q56

Keywords: Multi-regional input-output model, carbon rucksacks, trade, carbon accounting methods, consumption-based emissions

## 1. Introduction

Policies in the OECD countries aimed at the reduction of nationally produced $CO_2$ emissions may result in the relocation of emission intensive industries into countries with less stringent policies. This relocation often not only leads to a simple relocation of emissions, but to an absolute increase of emissions due to less advanced technology in the countries of relocation, thus not contributing to climate change mitigation at all. Additionally, newly emerging economies without harsh climate change policies such as China and India produce more goods for domestic consumption as well as for exports than ever before. The OECD countries quintupled their import volume from these countries between 1995 and 2005. This process of relocating industrial plants as well as the total increase of imports from countries without stringent climate change policies is called carbon leakage.

In light of the UN Climate Change Conference in December 2009, where – once again – no internationally binding agreement on the reduction of greenhouse gas (GHG) emissions was reached, it may also be necessary to rethink the basis on which national reduction targets for international emission abatement agreements are calculated. Currently, within most policy contexts such as the Kyoto protocol or the EU Emission Trading Scheme (EU ETS), GHG emissions associated with a country are calculated on the basis of emissions produced within the country's geographical borders. This is the most direct approach and the easiest from a data collecting point of view. Emissions are simply allocated to the country in which they are generated.

There is a major drawback of this approach, which is mainly emphasized by emerging economies and least developed countries. The critique is that most of the emissions produced in emerging economies and least developed countries are due to export demand. In other words, emissions within these countries are generated when producing products for exports to developed countries. International trade, its carbon rucksacks and the resulting carbon leakage should therefore be considered in the allocation of emissions to countries. A consumption-based accounting of emissions is often considered to be fairer, as it is not the producing, but the consuming country's demand that drives GHG emissions. Calculating consumption-based emissions though is more involving than calculating production-based emissions. The Global Resource Accounting Model (GRAM) allows for calculating these consumption-based emissions for 53 countries and 2 regions, disaggregated into 48 sectors, for the years between 1995 and 2005.

The paper is organized as follows: The first section gives a short discussion on territorial versus economy wide carbon accounting, followed by a short overview on existing methodologies of economy wide, i.e. consumption-based, carbon accounting. The two main approaches are life-cycle analysis (LCA) and input-output (IO) models. The next section gives a detailed model description of GRAM, and the last section shows prospective results.

## 2. Literature

Anthropogenic sources of carbon emissions are a major cause of climate change. These emissions originate from various sources like the combustion of fossil fuels in industry or households, but also from indirect sources like carbon emissions caused by land use change. Furthermore there are many natural sources and sinks of carbon emissions which, nevertheless, are affected by human interaction to a greater or lesser extent.

An accounting scheme may include all these sources or only selected ones. According to the scope of an accounting system it can be differentiated into full and partial carbon accounting as shown in Table 1. Emission flows can be divided by their sources into emissions from anthropogenic sources (direct and indirect emissions caused by human action), and from natural sources (natural sources and sinks of carbon emissions).

Apart from this, accounting systems are classified according to the concept on which they are based. In the majority of cases the accounting of carbon emissions nowadays is production based (territorial accounting); i.e. all emissions emitted on national territory are considered. This type of accounting is used e.g. under the Kyoto Protocol and the EU Emissions Trading Scheme. On the contrary, a consumption-based approach to carbon accounting aims to account for all carbon emissions released along the production chain of all domestically consumed products.

**Table 1: Classification scheme of approaches to carbon accounting**

| **Scope**<br>**Concept** | **Full** | **Partial** |
|---|---|---|
| **Production-based** | Full territorial carbon accounting | Partial territorial carbon accounting |
| **Consumption-based** | Full economy-wide carbon accounting | Partial economy-wide carbon accounting |

The concept of territorial accounting as used under Kyoto is a straight forward approach to account for carbon emissions. It is characterized by clear system boundaries and good data availability. However, reductions in the levels of territorial carbon emission can occur due to many reasons. One of them is the relocation of industries to other countries. A national policy designed to reduce carbon emissions pursuing a territorial approach of carbon accounting thus may well result in an increase of global emissions.

Therefore, in order to assess world-wide environmental consequences related to production and consumption of a specific country or world region, it is necessary to take international trade aspects fully into account. Only thereby possible shifts of environmental burden, resulting from changing global patterns of production, trade and consumption, can be illustrated.

A number of studies examined the distribution of environmental pressures between different world regions due to the economic specialisation in the international division of labour, applying methods of physical accounting and environmental-economic modelling. Several studies found empirical evidence for increasing externalisation of environmental burden by industrialised countries through trade and increasing environmental intensity of exports of non-OECD countries (see, for example, Ahmad and Wyckoff, 2003; Hertwich and Peters, 2009; Lenzen et al., 2004; Nakano et al., 2009; Nansai et al., 2008; Nijdam et al., 2005; Peters et al., 2004; Peters and Hertwich, 2006, 2008 a,b; Turner et al. 2007; Wiedmann, 2009 a,b; Wiedmann et al., 2007,2008).

These findings become particularly relevant, as the externalisation of environmental burden through international trade might be an effective strategy for industrialised countries to maintain high environmental quality within their own borders, while externalising the negative environmental consequences of their consumption processes to other parts of the world (see, for example, Ahmad and Wyckoff, 2003; Giljum and Eisenmenger, 2004; Muradian and Martinez-Alier, 2001; Tisdell, 2001; Weisz, 2006).

This global environmental responsibility is increasingly addressed by environmental policy strategies of the European Union and the OECD. One of the overall objectives of the renewed EU Sustainable Development Strategy (EU SDS) is to “actively promote sustainable development worldwide and ensure that the European Union’s internal and external policies are consistent with global sustainable development and its international commitments” (European Council, 2006, p. 20).

There are two main approaches for the calculation of consumption-based carbon emissions: life-cycle analysis (LCA) and input-output models (IO). LCA considers individuals products, and calculates the emissions embodied in each component of the product, based on emission coefficients. These coefficients though are based on an involving data sampling and generation process, so that until now carbon rucksacks were calculated only for selected products. For the approximating of carbon rucksacks of the trade of nations, the second methodology, input-output models, is better suited.

There are basically two ways of modeling $CO_2$-emissions in input-output models. The first, suggested by Leontief (1970) and followed up by Lenzen and various authors (see for instance Lenzen et al. 2004, 2007), is to add one row to the Leontief matrix for the “pollution sector” which “provides” pollution to all other sectors, having total pollution as the row sum. The other one, used by for example Peters and Hertwich and co-authors and Wiedmann and co-authors, is to multiply the Leontief inverse with a pollution-intensity coefficient matrix. Both methods are shortly explained in the next section.

## 3. MRIO theory

As we are not only interested in the pollution produced in one country, but rather in pollution ($CO_2$-emissions) embodied in international trade, we need to look at multi-region input-output

(MRIO) models. In this paper we will concentrate on the former case having one multi-region technical coefficient matrix.

The basic equation of each input-output model with technical coefficient (inter-industry requirements) matrix $A$, final demand $y$, and sectoral output $x$ is:

$$Ax + y = x$$

$$y = (I - A)x$$

$$x = (I - A)^{-1} y$$

This easily extends to the multiregional input-output model with $C$ regions in its most simple form:

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1C} \\ x_{21} & x_{22} & \cdots & x_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ x_{C1} & x_{C2} & \cdots & x_{CC} \end{pmatrix} = \begin{pmatrix} I - A_{11} & -A_{12} & \cdots & -A_{1C} \\ -A_{21} & I - A_{22} & \cdots & -A_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{C1} & -A_{C2} & \cdots & I - A_{CC} \end{pmatrix}^{-1} \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1C} \\ y_{21} & y_{22} & \cdots & y_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ y_{C1} & y_{C2} & \cdots & y_{CC} \end{pmatrix},$$

where $x_{ij}$ is the sectoral output in country $i$ to satisfy the demand in country $j$, $A_{ij}$ are the inter-regional input-output coefficient matrices, and $y_{ij}$ is the consumption demand of country $j$ for country $i$'s products.

The literature differentiates between three cases or stages of multi-regional pollution analysis using input-output models:

1. unilateral trade, using domestic $CO_2$ intensities for all imports
2. unilateral trade, different $CO_2$ intensities of imports according to exporting countries (all, or only major trading partners)
3. multilateral perspective with feedbacks of domestic countries.

For each of these cases the Leontief matrix is different. In the first two cases all but the column of matrices corresponding to the country of interest and the diagonal of the Leontief matrix are set to zero, while the last case uses the full matrix. Both the "pollution sector" and the pollution intensity matrix approach can be applied to any of the three cases.

*"Pollution sector"*

The approach of introducing an additional row and column, the "pollution sector", to the conventional input-output matrix, was first developed by Leontief (1970). In a two sector example he adds to the two equation system ( $(I - A)X = Y$ )

$$a_{11}X_1 + a_{12}X_2 \qquad = Y_1$$

$$a_{21}X_1 + a_{22}X_2 \qquad = Y_2$$

equation

$$a_{31}X_1 + a_{32}X_2 - \overline{X}_3 = 0$$

with coefficients $a_{31}$ and $a_{32}$ representing the amount of pollution per unit of output of the sectors *1* and *2*, respectively, and $\overline{X}_3$ being the total, so far unknown, quantity of pollution produced within the economy. The resulting Leontief matrix is:

$$(I - A^*) = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & -1 \end{pmatrix}.$$

The system can now also be written in the usual matrix notation, $X^* = (I - A^*)^{-1} Y$, the * representing the matrices including the "pollution sector", and solved accordingly. According to Leontief (1970) it does not make any difference, if the original IO system without the third row is solved first and $CO_2$ emissions are calculated on that basis from the third equation or if the complete system, including the pollution sector row, is solved simultaneously. He does not extend his analysis to a multi-country case. For more than one country there would also be only one additional column and one additional row in the total Leontief matrix. Though, in this case it does make a difference whether or not the coefficients of the "pollution sector" are included in the matrix inversion. It does not make a difference for total emissions, which correspond to $\overline{X}_3$ in the one country case, but for the emissions per country.[1]

*Pollution-intensity coefficient matrix*

GRAM is based on the second MRIO approach, which uses a pollution intensity coefficient matrix. For a simple one-country IO model, the pollution generated when producing output $x$ is denoted as

$$P = Ex = E(I - A)^{-1} y,$$

with $E$ being a matrix of emission intensities of each pollutant for each sector. In the literature $K$ different resources or pollutants are differentiated, resulting in resource or pollution intensity matrices $E_c$ of size ($K \times N$) for all countries $c$ for $N$ sectors. Using these, we can calculate direct resource-use or pollution intensities of each country's sectoral outputs, where $p_{ij}$ is the pollution associated with production in country $i$ to satisfy country $j$'s final demand:

$$\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1C} \\ p_{21} & p_{22} & \cdots & p_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ p_{C1} & p_{C2} & \cdots & p_{CC} \end{pmatrix} = \begin{pmatrix} E_1 & 0 & \cdots & 0 \\ 0 & E_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_C \end{pmatrix} \begin{pmatrix} I - A_{11} & -A_{12} & \cdots & -A_{1C} \\ -A_{21} & I - A_{22} & \cdots & -A_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{C1} & -A_{C2} & \cdots & I - A_{CC} \end{pmatrix}^{-1} \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1C} \\ y_{21} & y_{22} & \cdots & y_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ y_{C1} & y_{C2} & \cdots & y_{CC} \end{pmatrix}$$

Pollution embodied in exports and imports of a country are then simple row and column sums in matrix *P*, without the entry on the diagonal, which represents domestic pollution for

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[1] For a proof of this statement, please contact the author to get the 2-country example.

domestic consumption, respectively. More details of this approach will be given in the following section on GRAM.

## 4. The Global Resource Accounting Model GRAM

The **G**lobal **R**esource **A**ccounting **M**odel (GRAM) is a multi-regional input-output (MRIO) model, covering 53 countries and 2 regions and 48 sectors per country/region. See Table A1 in the Appendix for a list of countries explicitly modelled in GRAM. It calculates historic data of $CO_2$ emissions by consuming country for the years between 1995 and 2005 that do not yet exist in this form for a large group of countries and sectors. Furthermore, the results are detailed in such a way that the countries of origin are determined within the model.

Wiedmann (2009, p.1975) states that "…MRIO models – once fully developed – will be particularly suitable in the future to estimate the Ecological Footprints of imports and exports of nations with the possibility to track their own origin via inter-industry linkages, international supply chains and multi-national trade flows". "Once fully developed" refers to three major problems or drawbacks of the approach (Wiedmann et al., 2006; Lenzen et al., 2004; Peters and Hertwich, 2006a): The first problem to mention is data availability concerning the flows of traded goods. We will use each country's import IO table combined with national trade shares from the bilateral trade matrices as an approximation for this data.

The second problem also concerns data availability as the data might come from different sources and does not perfectly fit together. But since input-output tables and bilateral trade data used by GRAM are both provided by the OECD, they are compatible. The last problem is summarized under the heading "computability", again referring to difficulties arising from un-harmonized data, but also to the computational complexity of the system if several countries are involved. The Leontief matrix for 54 countries plus the rest of the world and 48 sectors consists of 2640 rows and 2640 columns. Inverting such a matrix takes a very long time if no appropriate software and calculation capacities are available. The GWS model system is based on C++, the INFORUM software and also own extensions of both, making it possible to invert this matrix in reasonable time.

### *Theoretical foundation of GRAM*

GRAM is a "true" MRIO model, as defined by Giljum et al. (2007), incorporating one global inter-industry requirements matrix $A$. It therefore differs from the widely used linked single-region form of MRIO models. These are models that include one IO model per country, which is solved separately from the others, and then linked to the other country models via international trade. The GRAM model implicitly includes international trade in the inter-industry requirements matrix, which is calculated from monetary input-output tables and bilateral trade data of the OECD. The central equation of the model is

$$\underbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1C} \\ p_{21} & p_{22} & \cdots & p_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ p_{C1} & p_{C2} & \cdots & p_{CC} \end{pmatrix}}_{P} = \underbrace{\begin{pmatrix} E_1 & 0 & \cdots & 0 \\ 0 & E_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_C \end{pmatrix}}_{E} \underbrace{\begin{pmatrix} I-A_{11} & -A_{12} & \cdots & -A_{1C} \\ -A_{21} & I-A_{22} & \cdots & -A_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{C1} & -A_{C2} & \cdots & I-A_{CC} \end{pmatrix}^{-1}}_{(I-A)^{-1}} \underbrace{\begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1C} \\ y_{21} & y_{22} & \cdots & y_{2C} \\ \vdots & \vdots & \ddots & \vdots \\ y_{C1} & y_{C2} & \cdots & y_{CC} \end{pmatrix}}_{y}.$$

The pollution matrix $P$ (the main result of this model) and the final demand matrix $y$ are composed of 55×55 vectors $p_{ij}$ and $y_{ij}$, respectively. Each of these vectors has 48 entries corresponding to the sectoral classification of the OECD input-output data. $y_{ij}$ represents the final demand in each sector of country $j$ directed at the production of country $i$. $p_{ij}$ is the pollution in each sector associated with production in country $i$ to satisfy demand in country $j$. The matrices $P$ and $y$ are of size 2640×55. The matrix $A$ includes inter-industry requirements between all sectors in all countries. It consists of 55 by 55 48×48 matrices $A_{ij}$. Hence, the total dimension of this matrix is 2640×2640. The matrices $A_{ii}$ correspond to inter-industry requirements within a country, and the matrices $A_{ij}$, for $j \neq i$, correspond to inter-industry requirements of exporting country $i$ and importing country $j$. The submatrices $A_{ij}$ for $j \neq i$ are calculated as follows: From the bilateral trade matrices we can calculate the share of each exporting country in the imports of all other countries. These shares are then multiplied with the import input-output (IO) matrix, leaving us with $C-1=54$ import IO matrices for each country. The coefficient matrices $A_{ij}$ are then easily calculated by dividing each entry by the corresponding total output of the sector.

The matrix $E$ (2640×2640) is a matrix including emission intensities on the diagonal (tons $CO_2$ per USD). The emission intensity matrix $E$ can also be replaced by a matrix containing raw material input requirements. The pollution matrix $P$, with its subvectors $p_{ij}$, then has to be interpreted as material extracted in country $i$ to produce the products consumed in country $j$.

The final results of the model are manifold: trade balances for all countries, including embodied emissions or raw materials, embodied emissions in domestic production and consumption, direct and indirect emission/raw material intensities.

### *Data*

The main data-sources are the OECD Input-Output Database (2009 release, more information can be found in Yamano and Ahmad, 2006), the OECD Bilateral Trade Database (2008), and $CO_2$ emission data and energy balances of the IEA (2008a, b).

The first step in GRAM is to calculate emission intensities. For that we use the IEA emissions data of the sectoral approach. These data are available for four energy carriers, see Table 2. GRAM takes emission data from the rows highlighted in grey, and then uses the energy balances (EB) of the IEA, which are also available for the different energy carriers, to split the emissions among the different economic sectors of the energy balances. For that we assume that the emission factors of each carrier are the same across sectors. Emissions are divided among the EB sectors for each carrier, and then added across the carriers to get one value per sector. That is, for example, for each energy carrier row 5 of the emission data (“manufacturing industries and construction”) is divided among rows 27 to 39 of the EB data:

$$\mathrm{EmissionsEB}_{\mathrm{i}}^{\mathrm{ET}} = Emissions_5^{ET} \frac{EB_i^{ET}}{\sum_{j=27}^{39} EB_j^{ET}}, \quad \forall i = 27,...,39.$$

**Table 2: Example - Austrian emissions in 2000, Mill. t $CO_2$**

| | Total | Coal & Peat | Gas | Oil | Others |
|---|---|---|---|---|---|
| 1 CO2 Sectoral Approach | 62,01 | 14,2 | 15,03 | 31,64 | 1,14 |
| 2 Main Activity Producer Electricity and Heat | 9,47 | 4,63 | 3,41 | 1,1 | 0,32 |
| 3 Unallocated Autoproducers | 3,69 | 1,76 | 1,16 | 0,62 | 0,16 |
| 4 Other Energy Industries | 6,02 | 3,92 | 0,64 | 1,47 | 0 |
| 5 Manufacturing Industries and Construction | 13,72 | 2,88 | 5,41 | 4,83 | 0,6 |
| 6 Transport | 16,66 | 0 | 0,53 | 16,13 | 0 |
| 7 of which: Road | 15,81 | 0 | 0 | 15,81 | 0 |
| 8 Other Sectors | 12,46 | 1,01 | 3,88 | 7,5 | 0,06 |
| 9 of which: Residential | 8,53 | 0,88 | 2,72 | 4,93 | 0 |
| 10 CO2 Reference Approach | 62,38 | 14,24 | 15,04 | 31,96 | 1,14 |
| 11 Diff. due to Losses and/or Transformation | 0,17 | -0,11 | 0 | 0,28 | 0 |
| 12 Statistical Differences | 0,2 | 0,15 | 0,01 | 0,04 | 0 |
| 13 Memo:International Marine Bunkers | 0 | 0 | 0 | 0 | 0 |
| 14 Memo:International Aviation | 1,63 | 0 | 0 | 1,63 | 0 |

*Source: IEA 2008c*

The next step is to allocate the emissions to the industries corresponding to the OECD Input-Output (IOT) sector classification. This is done according to the monetary structure of the energy supplying sector 8 (Coke, refined petroleum products and nuclear fuel) in the IOTs. These calculations result in emission vectors (*co2iot*) for each country, corresponding to the sectoral classification of the IOTs. Emission intensities are then calculated by dividing sectoral emissions by total sector output, given in row 10 of the primary inputs matrix (*PI*) of the IOT:

$$e[i] = \frac{co2iot[i]}{PI[10][i]}.$$

Linking emission intensities with the MRIO is then done by filling the diagonal entries of the $E_c$ matrices with the emission intensity vectors $e_c$ for each country $c$:

$$E_c = \begin{pmatrix} e_c[1] & 0 & \cdots & 0 \\ 0 & e_c[2] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e_c[48] \end{pmatrix}.$$

Because the $E$ matrices are diagonal, the results show the pollution generated in country $i$ due to final consumption demand in country $j$ in sectoral disaggregation, i.e. the $p_{ij}$’s are vectors of size 48. In the literature $E_c$ is mostly assumed to be a row vector, so that $p_{ij}$ are the scalars, representing the sum over all sectors of emissions produced in country *i* for satisfying country *j*'s demand.

The input-output tables (IOTs) of the OECD consist of 5 different matrices: the domestic inter-industry requirements matrix (*AR*, size 48×48), the domestic final demand matrix (*FD*, size 48×10), the imported inter-industry requirements matrix (*IMR*, size 48×48), the imported final demand matrix (*FDM*, size 48×10), and the primary inputs matrix (*PI*, size 10×48), including total sector output in row 10. *AR* and *IMR* are share matrices calculated from the original IOTs by dividing each entry by the corresponding total industry output in the primary inputs matrix (*PI[10][i]*).

The OECD IOTs are not available for all countries/regions modelled in GRAM. Further, they are available for at most three different years per country. Generally these years are 1995, 2000, and 2005, but there are deviations from this. Table A1 in the Appendix shows the data availability. The matrices for those countries for which no IOT is provided by the OECD are approximated by using the IOTs of neighbouring countries or countries with a similar economic structure, as shown in Table 3.

An important step in the calculations in GRAM is the approximations of IOTs for the years for which there is no IOT available. For that we have to distinguish two cases: first the case in which IO data for the starting or ending year (i.e. 1995 or 2005) are missing, and second the case of missing years between two years with existing IOTs. In the first case, it is simply assumed that the years before the first available year/after the last available year have the same structure, i.e. the same *A*R and *IMR* matrices, as the first/last available year. The final demand and primary input matrices, which are in million USD, are simply scaled proportional to the GDP change between the respective years. An example where both marginal years, 1995 and 2005, are missing is Switzerland. The IOT is only available for 2001, so that for example the final demand (*FD*) matrix for all other years is calculated as follows:

$$FD^t[i][k] = FD^{2001}[i][k] * \frac{GDP^t}{GDP^{2001}}$$

The same also holds for the primary inputs matrix.

In the second case, the matrices are calculated by linear interpolation between the two existing years, this is done for all three types of matrices, the interindustry requirements matrices *AR* and *IMR*, as well as the final demand matrices *FD* and *FDM* and the primary inputs matrices *PI*. Note that *FD*, *FDM*, and *PI* are then also scaled using the GDP ratio of the corresponding years:

Inter-industry requirements matrix *A* (*ARR*, *IMR*): $a_{ij}^{t+r} = \frac{(s-r)a_{ij}^{t} + ra_{ij}^{t+s}}{s}$

Final demand matrix $Y$ ($FD$, $FDM$): $y_{ij}^{t+r} = \dfrac{(s-r)\dfrac{GDPT^{t+r}}{GDPT^{t}}y_{ij}^{t} + r\dfrac{GDPT^{t+r}}{GDPT^{t+s}}y_{ij}^{t+s}}{s}$,

Again, the primary inputs matrix is calculated in the same way as the final demand matrices. A proof that the shares of a matrix still add up to 100 percent, i.e. that it is valid to use linear interpolation, is given in the appendix (A2). Note that by using linear interpolation, we assume that the inter-industry requirements change linearly between two given years. This is of course only an approximation of reality, but it is the best option we currently have.

**Table 3: Countries used for approximation**

| Country | Approximated by |
|---|---|
| Rest of the World | Argentina |
| Iceland | Norway |
| Bulgaria | Slovakia |
| Cyprus | Greece |
| Latvia | Poland |
| Lithuania | Poland |
| Malta | Greece |
| Romania | Slovakia |
| Honkong | Korea |
| Malaysia | Korea |
| Philippines | Korea |
| Singapore | Korea |
| Thailand | Korea |
| Chile | Brazil |
| OPEC | Indonesia |

The OECD provides bilateral trade data for 25 commodities and one service good. The commodity groups correspond almost completely to the sectoral disaggregation of the input-output matrices with two exceptions, the trade matrices do not differentiate between “Mining and quarrying (Energy)” and “Mining and quarrying (Non-Energy)” and the service matrix aggregates the service sectors 26 through 48 of the input-output classification.

Again, data availability has to be mentioned: even though there is data for every country modelled in GRAM, for some countries complete trade data is only available from 1996, 1997, or 1998 onwards. Table A1 in the appendix gives an overview on this. For the years before the first available year, we simply take import structures from the first available year.

## 5. Results

GRAM allows for the calculation of aggregated indicators of production versus consumption of $CO_2$ emissions of countries and world regions, taking into account emissions that occur along the international production chains. Thereby, comprehensive trade balances of embodied $CO_2$ emissions for individual countries or regions can be calculated and the main net-importers and net-exporters of $CO_2$ emissions in the world economy can be identified. GRAM results thus show the extent to which a country’s final demand is responsible for emissions produced abroad. These types of calculations are the empirical basis for the discussion as to whether producer or consumer countries are responsible for related environmental impacts.

The main output of GRAM is the 2640x55 pollution matrix *P*. It can easily be aggregated into a 55x55 $CO_2$ trade matrix, displaying the $CO_2$ exporting countries in the rows and the $CO_2$ importing countries in the columns. From this matrix it is straight forward to calculate carbon trade balances for all countries. Imports are column sums, minus the diagonal element, and exports are row sums, again minus the diagonal element. The entries on the diagonal are the emissions that are produced and consumed within the same country. See Table 5 for an 3-region version of this matrix. The countries are aggregated into three groups of countries, the OECD countries, a group of newly emerging economies, Brazil, Russia, India, China including Hong Kong and Taiwan, South Africa, and Argentina (in the following called BRICSA), and the rest of the world (RoW).

### *Carbon rucksacks*

Table 4 shows carbon balances of the US, Japan, France, Germany, the UK and the OECD as a whole on the one hand, and India, Russia, China, and all non-OECD countries on the other hand. The carbon trade balances directly show production-based carbon emissions (in row 4: domestic production) and consumption-based carbon emissions (in row 5: domestic consumption). The group of countries in the left part of the table are $CO_2$ net importers, which means that the sum of goods they consume has higher embodied $CO_2$ emissions than the sum of goods they produce. The non-OECD countries are generally net exporters of $CO_2$, which means that due to production processes in these countries more $CO_2$ is emitted than the $CO_2$ embodied in the products they consume.

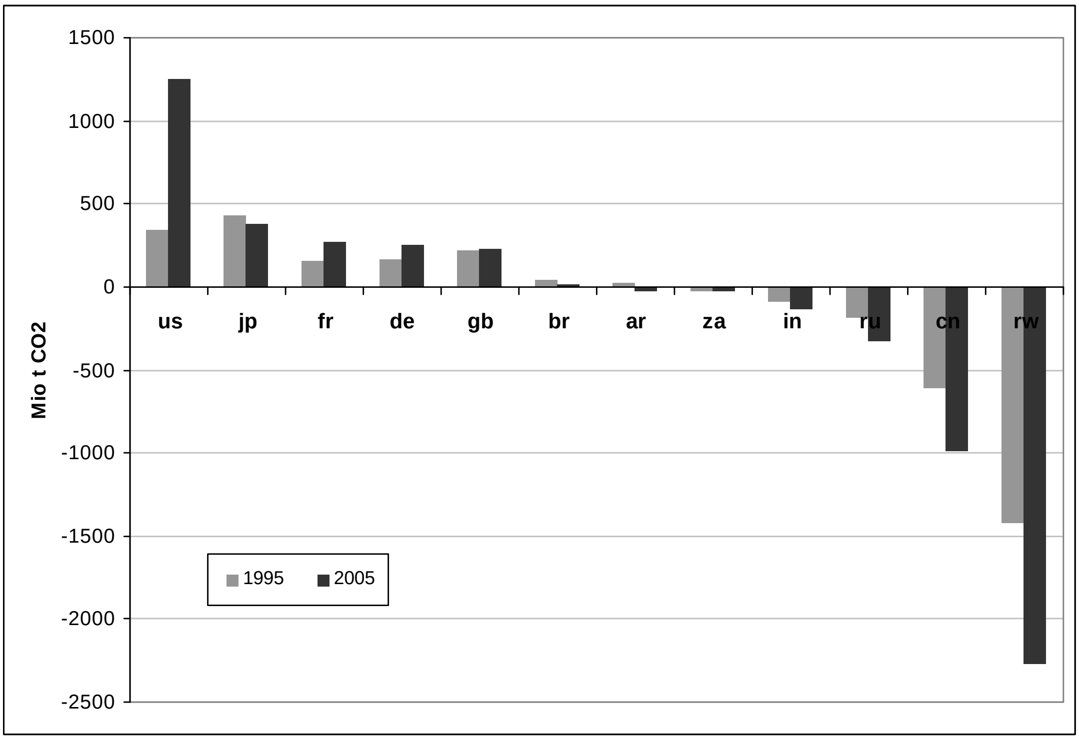
**Table 4: Carbon balances of selected countries**

| *in mio t CO2* | US | Japan | France | Germany | UK | OECD | India | Russia | China | on-OECD |
|---|---|---|---|---|---|---|---|---|---|---|
| **1995** | | | | | | | | | | |
| Domestic Production & Consumption | 3709 | 871 | 181 | 518 | 309 | 9009 | 588 | 680 | 2032 | 7511 |
| Exports | 460 | 107 | 78 | 176 | 102 | 292 | 131 | 221 | 727 | 2281 |
| Imports | 801 | 537 | 234 | 338 | 325 | 2281 | 42 | 35 | 120 | 292 |
| Domestic Production | 4170 | 978 | 258 | 694 | 411 | 9301 | 718 | 901 | 2759 | 9792 |
| Domestic Consumption | 4510 | 1409 | 415 | 856 | 633 | 11290 | 630 | 715 | 2152 | 7803 |
| Imports - Exports | 340 | 431 | 157 | 162 | 222 | 1989 | -88 | -186 | -607 | -1989 |
| Net-importer (I) / Net-exporter (E) | I | I | I | I | I | I | E | E | E | E |
| **2005** | | | | | | | | | | |
| Domestic Production & Consumption | 4295 | 859 | 183 | 414 | 329 | 10229 | 886 | 669 | 3092 | 10428,9 |
| Exports | 423 | 211 | 86 | 232 | 157 | 434 | 277 | 460 | 1357 | 3987 |
| Imports | 1678 | 592 | 361 | 488 | 389 | 3987 | 142 | 130 | 366 | 434 |
| Domestic Production | 4719 | 1070 | 269 | 645 | 486 | 10663 | 1163 | 1129 | 4449 | 14416 |
| Domestic Consumption | 5973 | 1450 | 544 | 902 | 718 | 14216 | 1028 | 799 | 3459 | 10863 |
| Imports - Exports | 1255 | 380 | 275 | 257 | 232 | 3553 | -136 | -330 | -990 | -3553 |
| Net-importer (I) / Net-exporter (E) | I | I | I | I | I | I | E | E | E | E |

Figure 1 shows the largest net importers of $CO_2$ on the left hand side and the BRICSA countries on the right. Japan (jp) was the country with the highest net $CO_2$ imports in 1995, closely followed by the US. Until 2005 the US almost quadrupled its $CO_2$ net imports, while Japan was able to decrease these imports by 12% compared to 1995. It was still the second largest $CO_2$ net importer in 2005, followed by France (fr), Germany (de), and the United Kingdom (gb). The UK was ranked third in 1995, while France and Germany had about the same net trade flows of $CO_2$, and were on ranks four and five.

From the BRICSA countries, both Argentina and Brazil were $CO_2$ net importers in 1995, but by 2005 Argentina was the largest net exporter in South America. The three largest $CO_2$ exporters in 1995 and 2005 were China (cn), Russia (ru), and India (in). While in 1995, China's carbon trade balance deficit (net exports) was twice as much as the US' carbon trade balance surplus (net imports), in 2005 US carbon net imports already reached the level of Chinese plus Russian net exports of embodied $CO_2$. About half of all world wide net exports come from RoW, which includes all African countries except South Africa and those belonging to the OPEC, and some smaller South American, South East Asian, and Middle Eastern countries, which are all considered as least developed countries. The net exports from non-OECD countries to OECD countries amount to more than one quarter of total production-based $CO_2$ emissions in the non-OECD countries.

**Figure 1: Carbon net imports and net exports**



*Carbon leakage*

The term "carbon leakage" summarizes all processes which lead to an increase of $CO_2$ emissions abroad caused by activities that reduce or avoid domestic emissions. Carbon leakage does not only occur in its direct form of relocating domestic production into other countries, but also indirectly. Indirect forms of carbon leakage are changing trade patterns: on the one hand satisfying increasing domestic demand by increasing imports (and not by increasing domestic production), and on the other hand a (e.g. price-induced) switch from goods produced domestically with relatively clean technologies to the same goods produced abroad with relatively more polluting technologies.

The two matrices on top of Table 5 are carbon trade matrices for the three regions OECD, BRICSA, and RoW. Entry (i,j) reflect exports from region i to region j, in accordance with pollution matrix *P* in the model. While in 1995 80% of the $CO_2$ emission associated with OECD consumption was produced within the OECD countries, this number decreased to 72% in 2005. That means that carbon rucksacks on imports to the OECD did not only increase due to an increase in total trade volume (which is reflected in increasing carbon imports and exports), but also relative to $CO_2$ emission associated with both production and consumption within the OECD countries. This increase in net-imports of $CO_2$ to OECD countries, highlighted in grey in Table 5, shows the amount of carbon leakage that took place between 1995 and 2005. In 1995 the OECD net-imports of $CO_2$ from the BRICSA countries and the rest of the World (RoW) amount to 1989 Mio t. This number almost doubles until 2005 to 3553 Mio t. 60% of this increase stems from imports from the BRICSA countries. That means that 60% of total carbon leakage from the OECD occurred in only six countries. The OECD countries are responsible for one quarter of the emissions produced in the BRICSA countries in 2005. This number was about 15% in 1995. The fraction of emissions in RoW induced by consumption in OECD countries hardly changes between 1995 and 2005, from 29.5% to 30.5%. This means that the OECD countries consume increasingly more $CO_2$ embodied in goods that are produced in the BRICSA countries, and hence, carbon leakage mainly occurs between the OECD and the BRICSA countries and not so much within the remaining RoW countries.

**Table 5: Carbon trade between OECD, BRICSA and the rest of the world (RoW)**

**1995**

| *Mio t CO2* | **OECD** | **BRICSA** | **RoW** | Production |
|---|---|---|---|---|
| **OECD** | 9009 | 154 | 138 | 9301 |
| **BRICSA** | 882 | 3930 | 239 | 5051 |
| **RoW** | 1399 | 164 | 3178 | 4741 |
| Consumption | 11290 | 4248 | 3555 | |

| *% of consumption* | **OECD** | **BRICSA** | **RoW** | OECD Net-Imports *Mio t CO2* |
|---|---|---|---|---|
| **OECD** | 80% | 4% | 4% | |
| **BRICSA** | 8% | 93% | 7% | 728 |
| **RoW** | 12% | 4% | 89% | 1261 |

**2005**

| *Mio t CO2* | **OECD** | **BRICSA** | **RoW** | Production |
|---|---|---|---|---|
| **OECD** | 10229 | 251 | 183 | 10663 |
| **BRICSA** | 1906 | 5591 | 304 | 7800 |
| **RoW** | 2081 | 540 | 4204 | 6826 |
| Consumption | 14216 | 6381 | 4692 | |

| *% of consumption* | **OECD** | **BRICSA** | **RoW** | OECD Net-Imports *Mio t CO2* |
|---|---|---|---|---|
| **OECD** | 72% | 4% | 4% | |
| **BRICSA** | 13% | 88% | 6% | 1655 |
| **RoW** | 15% | 8% | 90% | 1898 |

## 6. Discussion

GRAM calculates $CO_2$ emissions, for the first time by consuming country for 53 countries and two regions for the years between 1995 and 2005. GRAM uses a MRIO modelling approach with carbon intensities. The main outcome of the model, one carbon trade matrix (of size 2640x55) per year, is disaggregated according to 48 sectors of origin per country for 55 countries and regions with destinations being the final demand of the 55 countries and regions. From this matrix carbon trade balances, consumption- and production-based carbon emissions can be calculated. It is further possible to extract direct carbon intensities at sector level for each of the modelled countries.

A current limitation of GRAM is induced by data gaps, especially for the two regions OPEC and Rest of the World, where trade data gaps had to be completed within the model using simple assumptions. Further, trade in services data is only readily available for the years from 2000 onwards, hence the structure from 2000 was used to approximate trade in services for the years 1995 to 1999. An approximation of the input-output structure was made for 13 countries and the two regions, as no OECD input-output tables were available. These data gaps will be filled in future versions of GRAM as soon as better data become available.

A possible extension of the present GRAM results is Structural Path Analysis (SPA). By using SPA in environmentally extended input-output models, production paths with the highest embodied $CO_2$ emissions can be identified, as is for example done in Lenzen et al. (2007), Minx et al. (2008), Peters and Hertwich (2006c), Wood (2008). SPA is computationally involving and can only be made for a few destination countries at a time. The algorithm we use for GRAM is based on Peters and Hertwich (2006c). Results for Austria show that the energy sector has highest direct and indirect emissions, followed by the transport sector, machinery and equipment, and construction. Analyses for other countries will follow.

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## Appendix A1

### Table A1: Country coverage in GRAM and data availability

| no | | Bezeichnung OECD | Startjahr BTD | Trade in services (OECD) | IO (OECD 2009) | IEA/Energy | IMF |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| 1 | at | Austria | 1995 | x | **1995/2000/2004** | x | x |
| 2 | be | Belgium | 1988 | x | **1995/2000/2004** | x | x |
| 3 | lu | Luxembourg | 1999 | x | **1995/2000/2005** | x | x |
| 4 | dk | Denmark | 1988 | x | **1995/2000/2004** | x | x |
| 5 | fi | Finland | 1988 | x | **1995/2000/2005** | x | x |
| 6 | fr | France | 1988 | x | **1995/2000/2005** | x | x |
| 7 | de | Germany | 1988 | x | **1995/2000/2005** | x | x |
| 8 | gr | Greece | 1988 | x | **1995/1999/2005** | x | x |
| 9 | ie | Ireland | 1988 | x | **1998/2000** | x | x |
| 10 | it | Italy | 1988 | x | **1995/2000/2004** | x | x |
| 11 | nl | Netherlands | 1988 | x | **1995/2000/2005** | x | x |
| 12 | pt | Portugal | 1988 | x | **1995/2000/2005** | x | x |
| 13 | es | Spain | 1988 | x | **1995/2000/2004** | x | x |
| 14 | se | Sweden | 1988 | x | **1995/2000/2005** | x | x |
| 15 | gb | United Kingdom | 1988 | x | **1995/2000/2003** | x | x |
| 16 | cz | Czech Republic | 1993 | x | **2000/2005** | x | x |
| 17 | hu | Hungary | 1992 | x | **1998/2000/2005** | x | x |
| 18 | pl | Poland | 1992 | x | **1995/2000/2004** | x | x |
| 19 | sk | Slovak Republic | 1997 | x | **1995/2000** | x | x |
| 20 | tr | Turkey | 1989 | x | **1996//2002** | x | x |
| 21 | is | Iceland | 1988 | | | x | x |
| 22 | no | Norway | 1988 | x | **1995/2000** | x | x |
| 23 | ch | Switzerland | 1988 | | | x | x |
| 24 | ca | Canada | 1988 | x | **1995/2000** | x | x |
| 25 | mx | Mexico | 1990 | x | **2003** | x | x |
| 26 | us | United States | 1990 | x | **1995/2000/2005** | x | x |
| 27 | jp | Japan | 1988 | x | **1995/2000/2005** | x | x |
| 28 | kr | Korea | 1994 | x | **2000** | x | x |
| 29 | au | Australia | 1988 | x | **1998/99/2004/05** | x | x |
| 30 | nz | New Zealand | 1989 | x | **1995/96/2002/03** | x | x |
| 31 | bg | Bulgaria | | | | x | x |
| 32 | cy | Cyprus | | | | x | x |
| 33 | ee | Estonia | 1995 | | **1997/2000//2005** | x | x |
| 34 | lv | Latvia | | | | x | x |
| 35 | lt | Lithuania | | | | x | x |
| 36 | mt | Malta | | | | x | x |
| 37 | si | Slovenia | 1994 | | **2000/2005** | x | x |
| 38 | ro | Romania | | | | x | x |
| 39 | cn | China | 1992 | | **1995/2000/2005** | x | x |
| 40 | hk | Hong Kong, China | 1992 | x | | x | x |
| 41 | id | Indonesia | 1989 | | **1995/2000/2005** | x | x |
| 42 | in | India | 1988 | | **1993/94/1998/99** | x | x |
| 43 | my | Malaysia | 1989 | | | x | x |
| 44 | ph | Philippines | 1996 | | | x | x |
| 45 | sg | Singapore | 1989 | | | x | x |
| 46 | th | Thailand | 1988 | | | x | x |
| 47 | tw | Chinese Taipei | 1990 | | **1996/2001** | x | NSO |
| 48 | ar | Argentina | 1993 | | **1997** | x | x |
| 49 | br | Brazil | 1989 | | **1995/2000/2005** | x | x |
| 50 | cl | Chile | 1990 | | | x | x |
| 51 | za | South Africa | 2000 | | **1993/2000** | x | x |
| 52 | il | Israel | 1995 | | **1995** | x | x |
| 53 | ru | Russian (Federation ( | 1996 | x | **1995/2000** | x | x |
| 54 | op | OPEC excl. Indonesia | | | | x | x |
| 55 | rw | Rest of the world | | | | x | |

## Appendix A2: Mathematical proof of linear interpolation of matrices

Assume two matrices $A^t = \begin{bmatrix} a_{11}^t & a_{12}^t & a_{1n}^t \\ a_{21}^t & a_{22}^t & a_{2n}^t \\ & & \\ a_{n1}^t & a_{n2}^t & a_{nn}^t \end{bmatrix}$ and $A^{t+s} = \begin{bmatrix} a_{11}^{t+s} & a_{12}^{t+s} & a_{1n}^{t+s} \\ a_{21}^{t+s} & a_{22}^{t+s} & a_{2n}^{t+s} \\ & & \\ a_{n1}^{t+s} & a_{n2}^{t+s} & a_{nn}^{t+s} \end{bmatrix}$ with the properties that $\sum_{i=1}^{n} a_{ij}^{t} = 100$ and $\sum_{i=1}^{n} a_{ij}^{t+s} = 100$ from which you want to calculate a third matrix $A^{t+r} = \begin{bmatrix} a_{11}^{t+r} & a_{12}^{t+r} & a_{1n}^{t+r} \\ a_{21}^{t+r} & a_{22}^{t+r} & a_{2n}^{t+r} \\ & & \\ a_{n1}^{t+r} & a_{n2}^{t+r} & a_{nn}^{t+r} \end{bmatrix}$ for any 0 < r < s by linear interpolation: $a_{ij}^{t+r} = \frac{(s-r)a_{ij}^{t} + ra_{ij}^{t+s}}{s}$. We need to show that $\sum_{i=1}^{n} a_{ij}^{t+r} = 100$ as well:

$$\begin{aligned}
\sum_{i=1}^{n} a_{ij}^{t+r} &= \sum_{i=1}^{n} \frac{(s-r)a_{ij}^{t} + ra_{ij}^{t+s}}{s} \\
&= \frac{\sum_{i=1}^{n}\left((s-r)a_{ij}^{t} + ra_{ij}^{t+s}\right)}{s} \\
&= \frac{\sum_{i=1}^{n}(s-r)a_{ij}^{t} + \sum_{i=1}^{n} ra_{ij}^{t+s}}{s} \\
&= \frac{(s-r)\sum_{i=1}^{n} a_{ij}^{t} + r\sum_{i=1}^{n} a_{ij}^{t+s}}{s} \\
&= \frac{(s-r)\times 100 + r\times 100}{s} \\
&= \frac{100s - 100r + 100r}{s} = \frac{100s}{s} = 100.
\end{aligned}$$

This concludes the proof.